Mathematics (Subsidiary) B.Sc Part II Dated-16/04/21

Successive Differentiation

To find the nth differential coefficient of Sin(ax+b):

$$\text{let } y = \sin(ax+b)$$

$$\text{Then } y_1 = a\cos(ax+b)$$

$$= a\sin\left\{ax+b+\frac{\pi}{2}\right\} \quad \text{bcz } \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$$

$$y_2 = a^2\cos\left(ax+b+\frac{\pi}{2}\right)$$

$$= a^2\sin\left(ax+b+\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$= a^2\sin\left(ax+b+2\cdot\frac{\pi}{2}\right)$$

$$y_3 = a^3\cos\left(ax+b+2\cdot\frac{\pi}{2}\right)$$

$$= a^3\sin\left(ax+b+2\cdot\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$= a^3\sin\left(ax+b+3\cdot\frac{\pi}{2}\right)$$

Let us suppose that

$$y_n = a^n\sin\left(ax+b+n\cdot\frac{\pi}{2}\right) \quad \text{———— (1)}$$

differentiating (1), again with respect to x, we get

$$y_{n+1} = a^n\cos\left(ax+b+\frac{n\pi}{2}\right)\times a$$

$$= a^{n+1}\sin\left(ax+b+\frac{n\pi}{2}+\frac{\pi}{2}\right)$$

$$= a^{n+1}\sin\left(ax+b+(n+1)\frac{\pi}{2}\right)$$

Which is of the same from as (1) with n replaced by $(n+1)$.

Since (1) is true for $n = 1, 2, 3, \ldots$, therefore by the method of induction, it is true for all n.

Thus if $y = \sin(ax+b)$, then $y_n = a^n \sin\left(ax+b+\frac{n\pi}{2}\right)$

Particular case 1. If $b=0$, then $y = \sin ax$

$$\therefore y_n = a^n \sin\left(ax + \frac{n\pi}{2}\right)$$

2. If $a=1$, $b=0$ then $y = \sin x$.

$$\therefore y_n = \sin\left(x + \frac{n\pi}{2}\right).$$

Example – Find the nth differential coefficient of $e^x \sin^2 x$.

Solution:– Here $y = e^x \sin^2 x$

$$= e^x \frac{1-\cos 2x}{2}$$

$$= \frac{1}{2} e^x - \frac{1}{2} e^x \cos 2x$$

$\therefore$ From the formula

$$y_n = \frac{1}{2} e^x - \frac{1}{2} e^x (1^2 + 2^2)^{n/2} \times \cos\left(2x + n \tan^{-1}\frac{2}{1}\right)$$

$$= \frac{1}{2} e^x - \frac{1}{2} e^x (5)^{n/2} \cos(2x + n \tan^{-1} 2)$$